Static strings in Randall-Sundrum scenarios and the quark anti-quark potential

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Abstract

We calculate the energy of a static string in an AdS slice between two D3-branes with orbifold condition. The energy for configurations with endpoints on a brane grows linearly for large separation between these points. The derivative of the energy has a discontinuity at some critical separation. Choosing a particular position for one of the branes we find configurations with smooth energy. In the limit where the other brane goes to infinity the energy has a Coulombian behaviour for short separations and can be identified with the Cornell potential for a quark anti-quark pair. This identification leads to effective values for the AdS radius, the string tension and the position of the infrared brane. These results suggest an approximate duality between static strings in an AdS slice and a heavy quark anti-quark configuration in a confining gauge theory.

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Phenomenological models with extra dimensions have attracted much attention in last years. In particular, the Randall-Sundrum model [1] consists of a five dimensional anti-de Sitter (AdS) slice between two D3-branes with the standard model fields living on one of these branes. Here we calculate the world-sheet area of a static string in such a space. We find a potential energy with Coulomb like behaviour at short distances and linear confining behaviour for large distances. This energy can be identified with the Cornell potential for the strong interaction of a heavy quark anti-quark pair.

Understanding non perturbative aspects of strong interactions, like confinement and mass generation is a challenge for theoretical physicists. At high energies, QCD (Yang Mills SU(3) plus fermionic matter fields) has a small coupling constant and leads to a nice perturbative description of strong interactions. However at low energies the coupling is large, so the theory is non perturbative and one needs lattice calculations.

There are presently many indications that one can learn about this non perturbative regime of QCD from gauge/string dualities. A very important result relating SU(N) Yang Mills gauge theories with large N to string theory was obtained by 't Hooft[2] long ago. More recently Maldacena [3] discovered an exact gauge/string duality. The string theory is defined in a ten dimensional space which is the direct product of a five dimensional Anti-de Sitter space and a compact five dimensional space ($AdS_5 \times X^5$). The corresponding gauge theory is a superconformal large N Yang Mills theory on the four dimensional boundary of this space. This duality is known as AdS/CFT correspondence[3, 4, 5, 6]. A first connection between AdS/CFT and non conformal gauge theories was proposed by Witten in [7]. In this approach the AdS space accommodates a Schwarzschild black hole. This procedure introduces a scale, breaking conformal invariance, and can be used, for instance, to calculate glueball masses[8, 9, 10, 11, 12, 13, 14].

Important results on strong interactions have been obtained recently from phenomenological models inspired in the idea of gauge/string duality. Introducing an infrared cut off in the AdS space, Polchinski and Strassler obtained the high energy amplitude for glueball scattering at fixed angles from string theory[15]. They obtained also results for the structure functions of deep inelastic scattering using this framework [16]. This infrared cut off AdS space can be interpreted as an AdS slice, similar to that proposed in [1]. Using an AdS slice with boundary conditions the spectrum of glueballs [17, 18] and light baryons and mesons [19] was also obtained.

In a gauge theory, non perturbative aspects such as confinement can be studied with the help of Wilson loops $exp\{i \oint_C A^\mu dx_\mu\}$. For a static configuration it takes the form $exp\{-T\,E\}$ where E is the energy and T the time interval. Wilson loops can be used to calculate the potential energy of a heavy quark anti-quark pair and determine the behaviour with respect to the quark separation.

Gauge string/duality can be used to calculate Wilson loops. In the case of the AdS/CFT correspondence there is an exact duality and the Wilson loop of a heavy quark anti-quark pair in the superconformal large N gauge theory was calculated from the world-sheet area of a dual static string living in the AdS bulk[20, 21]. The string lies along a geodesic in the AdS bulk with endpoints on the boundary representing the quark and anti-quark positions. Its energy is proportional to the geodesic length and is a function of the quark anti-quark distance for an observer on the four dimensional boundary where the gauge theory is defined. In refs. [20, 21] it was shown that for the AdS$_5$ space the energy shows a purely Coulombian (non confining) behaviour, compatible with a conformal field theory. For calculations of Wilson loops following Witten's proposal[7] for a confining geometry see for instance[22, 23, 24, 25]. A discussion of Wilson loops associated with quark anti-quark potential in general spaces can be found in [26].

Here we are going to calculate the potential energy of a static string in an AdS slice with orbifold condition as in the Randall Sundrum model. In this model the standard model fields live on a $D3$-brane ($y = 0$) in a five dimensional space described by the metric

$$ds^2 = e^{-2|y|/R}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2 \tag{1}$$

where $\eta_{\mu\nu}$ is the four dimensional Minkoviski metric. The coordinate y is defined in the range $-y_c \le y \le y_c$, where y_c is a constant related to the compactification length and satisfies an orbifold condition corresponding to the identification: $(x, y) = (x, -y)$. This space corresponds to two identical AdS slices joined together with endpoints of the coordinate y identified. Note that we just need to consider one slice since the other is just a copy. It is convenient to describe the slice $y \ge 0$ using the coordinates x, r with $r = R\exp\{-y/R\}$ which implies

$$ds^2 = \Big(\frac{r^2}{R^2}\Big)(-dt^2 + d\vec{x}^2) + \Big(\frac{R^2}{r^2}\Big)dr^2\,. \tag{2}$$

with $r_2 = R\exp\{-y_c/R\} \le r \le r_1 = R$.

We want to calculate the potential energy associated with a static string with endpoints located on the standard model brane ($r_1 = R$). The string is described by the Nambu-Goto action

$$S = \frac{1}{2\pi\alpha'} \int d\sigma d\tau \sqrt{det(g_{MN}\partial_\alpha X^M \partial_\beta X^N)} \tag{3}$$

where g_{MN} is the five dimensional metric defined by (2). This action is proportional to the area of the world-sheet. For a static configuration the action is also proportional to the energy. Solving the classical equations of motion, that corresponds to finding the geodesics, one can estimate the energy from the string length.

Let us consider the geodesic between two points on the brane at $r_1 = R$ separated by a coordinate distance $\Delta x = L$. Since there is a brane at $r = r_2$, there will be two classes of geodesics solutions, as illustrated in fig. 1. For coordinate separations smaller than some critical value L_{crit} the geodesic does not reach r_2. In this case the geodesics reaches a minimum value r_0 of the coordinate r defined by the relation

$$L = \frac{2R^2}{r_0} I_1(R/r_0) \tag{4}$$

where $I_1(\xi)$ is the elliptic integral

$$I_1(\xi) = \int_1^\xi \frac{d\rho}{\rho^2\sqrt{\rho^4 - 1}} . \tag{5}$$

The critical coordinate separation $L = L_{crit}$ corresponds to $r_0 = r_2$. For $L > L_{crit}$ the geodesic reaches the second brane and contains a non null path along $r = r_2$. This happens because this region corresponds to a minimum of the metric thanks to the orbifold condition.

The string energy is proportional to the geodesic length. For $L \leq L_{crit}$ it can be calculated following refs. [26, 27] obtaining

$$E_{RS}^{(-)} = \frac{r_0}{\pi\alpha'} I_2(R/r_0) - \frac{r_0}{\pi\alpha'} \tag{6}$$

where we have subtracted the constant $R/\pi\alpha'$ for latter convenience and defined a second elliptic integral

$$I_2(\xi) = \int_1^\xi \Big[\frac{\rho^2}{\sqrt{\rho^4 - 1}} - 1\Big] d\rho . \tag{7}$$

The energy corresponding to the geodesics for $L \geq L_{crit}$ is calculated by adding the lengths of curves between $r = r_2$ and $r = R$ and the path along the brane at $r = r_2$ (see figure 1, curve **c**). After subtracting the same constant we obtain

$$E_{RS}^{(+)} = \frac{r_0}{\pi\alpha'} \Big(I_2(R/r_0) - I_2(r_2/r_0) \Big) - \frac{r_2}{\pi\alpha'} + \frac{r_2^2}{\pi\alpha' \, r_0} I_1(r_2/r_0). \tag{8}$$

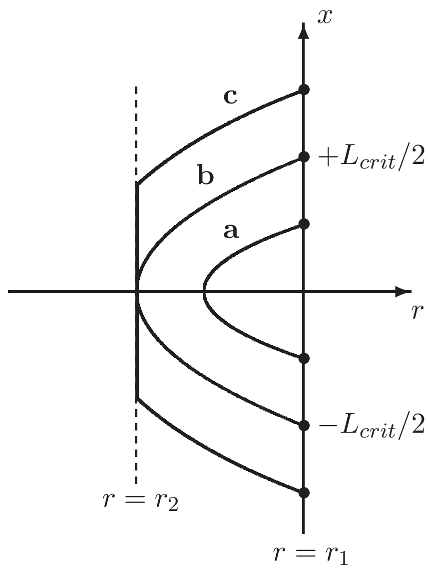

FIG. 1: Schematic representation of geodesics in the Randall Sundrum space. Curve **a** corresponds to a geodesic with $L < L_{crit}$, curve **b** to $L = L_{crit}$ and **c** to $L > L_{crit}$.

We note that for large values of L the energy increases linearly with L (confining behaviour). However, the energy is not a smooth function of L in this model. There is a discontinuity in the derivative at $L = L_{crit}$. See figure 2. This discontinuity decreases as we increase the value of r_2. If we take $r_2 \to R$ the discontinuity disappears. However this corresponds to placing the two branes at the same position and then having no AdS slice.

We can modify our initial set up to find an energy that varies smoothly with the endpoints separation. We consider the standard model brane at some $r_1 > R$ and the second brane at some r_2. We find out that the energy is smooth only if $r_2 = R$. So we take two AdS slices with orbifold identification each one described by the metric (2) but now with $r_2 = R \le r \le r_1$. In this case the relation between L and r_0 is

$$L \,=\, \frac{2R^2}{r_0}\, I_1(r_1/r_0) \tag{9}$$

The energy for $L \le L_{crit}$ can be defined subtracting the constant $r_1/\pi\alpha'$ in such a way that the energy is finite even in the limit $r_1 \to \infty$. We get

$$E^{(-)} \,=\, \frac{r_0}{\pi\alpha'}\Big[\, I_2(r_1/r_0) \,-\, 1\Big] \tag{10}$$

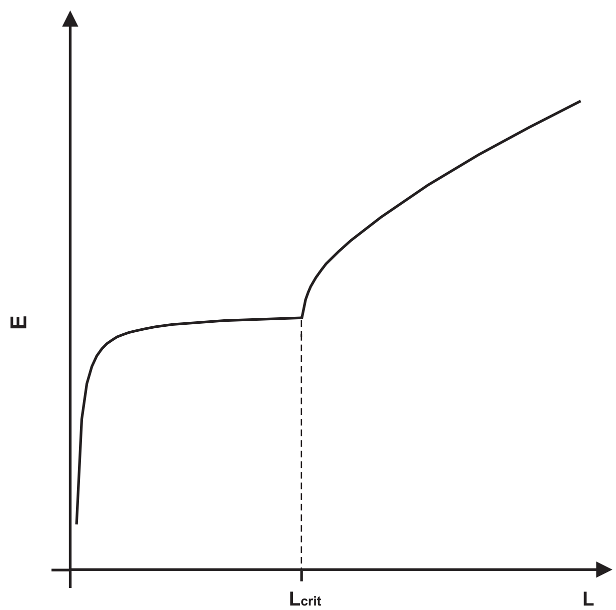

FIG. 2: Energy as a function of string end-points separation L in Randall Sundrum space $r_2 \leq r \leq R$ with discontinuous derivative at L_{crit}.

Substituting r_0 from eq. (9) we find

$$E^{(-)} = \frac{2R^2}{\pi\alpha'} \frac{I_1(r_1/r_0)}{L} \Big[I_2(r_1/r_0) - 1 \Big] . \tag{11}$$

For $L > L_{crit}$, again subtracting the constant $r_1/\pi\alpha'$, we obtain

$$E^{(+)} = \frac{r_0}{\pi\alpha'} \Big[I_2(r_1/r_0) - I_2(R/r_0) \Big] - \frac{R}{\pi\alpha'} + \frac{R^2}{\pi\alpha' r_0} I_1(R/r_0) \tag{12}$$

$$= \frac{2R^2}{\pi\alpha'} \frac{I_1(r_1/r_0)}{L} \Big[I_2(r_1/r_0) - I_2(R/r_0) \Big] - \frac{R}{\pi\alpha'} + \frac{L}{2\pi\alpha'} \frac{I_1(R/r_0)}{I_1(r_1/r_0)} , \tag{13}$$

where again we have substituted r_0 from eq. (9).

It is interesting to analyze the behaviour of these energies in the limit where the standard model brane is moved to infinity $r_1 \to \infty$. Noting that $I_1(\infty) \equiv C_1 = \sqrt{2}\pi^{3/2}/[\Gamma(1/4)]^2$ and $I_2(\infty) = 1 - C_1$, we find that the energy (11) behaves as $\sim 1/L$ and the energy (12) shows a linear behaviour for large values of L.

These asymptotic behaviors are analogous to those of the phenomenological Cornell potential for a heavy quark anti-quark pair [28, 29, 30]

$$E_{Cornell}(L) = -\frac{4}{3}\frac{a}{L} + \sigma L + const. . \tag{14}$$

where L is the quark separation, $a = 0.39$ and $\sigma = 0.182$ Gev^2. For a review see for instance [31, 32].

This kind of potential, with a linear and a Coulombian terms, has been also obtained from Wilson loops in other confining backgrounds [23, 24, 25].

The fact that our potential behaves asymptotically as the Cornell potential suggests that there is an approximate gauge/string duality relating strings in this AdS slice with a QCD like confining gauge theory. In particular, the world-sheet area of the static string considered here would be dual to the Wilson loop of a heavy quark anti-quark pair in the dual gauge theory. So the string endpoints correspond to the quark and anti-quark positions and L is the quark anti-quark distance from the point of view of the gauge theory.

Then we can identify the asymptotic behaviour of the string energy $E^{(-)}$ for small L with the Coulomb term of the Cornell potential by taking $a = 3C_1^2 R^2/2\pi\alpha'$. In the same way we can identify the asymptotic behaviour of $E^{(+)}$ for large L with the linear term of the Cornell potential by choosing $\sigma = 1/2\pi\alpha'$.

So the string energy as a function of end-point separation L in terms of the Cornell parameters is

$$E = \begin{cases} \dfrac{4a}{3C_1^2}\,\dfrac{I_1(r_1/r_0)}{L}\Big[\,I_2(r_1/r_0)\,-\,1\Big]\,, & L \le L_{crit} \\[2ex] \dfrac{4a}{3C_1^2}\dfrac{I_1(r_1/r_0)}{L}\Big[I_2(r_1/r_0) - I_2(R/r_0)\Big] - \sqrt{\dfrac{4\,a\,\sigma}{3\,C_1^2}} + \sigma L\,\dfrac{I_1(R/r_0)}{I_1(r_1/r_0)}\,, & L \ge L_{crit} \end{cases} \tag{15}$$

In the limit $r_1 >> R$, where according to eq. (9), $L_{crit} \to 2RC_1$ the energy takes the form

$$E = \begin{cases} -\dfrac{4a}{3\,L}\,, & L \le L_{crit} \\[2ex] -\dfrac{4a}{3L} + \dfrac{4a}{3C_1\,L}\Big[1 - I_2(R/r_0)\Big] - \sqrt{\dfrac{4\,a\,\sigma}{3\,C_1^2}} + \sigma L\,\dfrac{I_1(R/r_0)}{C_1}\,, & L \ge L_{crit} \end{cases} \tag{16}$$

Note that for $L \le L_{crit}$ the string does not reach the infrared brane. So, the potential takes the Coulombian form corresponding to a conformal theory as in AdS/CFT case. For $L >> R$ the energy takes the asymptotic linear form $E \sim \sigma\,L$ as in the Cornell potential. From the point of view of the gauge theory this implies confinement. In figure 3 we represent the energy for different brane positions $r_1 = nR$ and $r_2 = R$ fixed.

The identifications we have done for the energy in the Randall Sundrum scenarios (an AdS slice between two branes $r_2 = R \le r \le r_1$) determines an effective value for the AdS

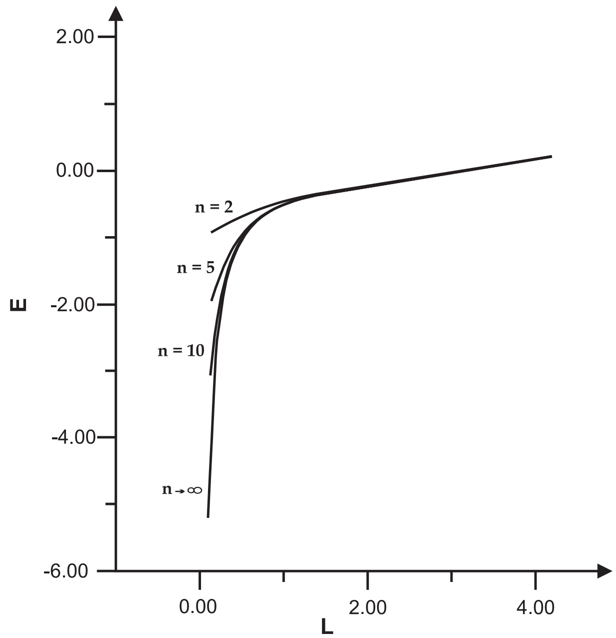

FIG. 3: Energy in GeV as a function of string end-points separation L in GeV^{-1} for AdS slices with $r_1 = nR$ and $r_2 = R$. For $n \to \infty$ the energy behaves as the Cornell potential eq. (14).

radius in this phenomenological model in terms of the Cornell parameters:

$$R = \sqrt{\frac{a}{3\sigma C_1^2}} = 1.4 \ \mathrm{GeV}^{-1} \ . \tag{17}$$

This corresponds to an effective energy scale of 0.71 GeV consistent with a QCD scale.

Concluding, we calculated static string energies in AdS slices with orbifold condition and found smooth energies when the infrared brane is located at $r_2 = R$. In the limit where the standard model brane goes to infinity the potential energy is identified with the Cornell potential for a heavy quark anti-quark pair. This fact suggests a duality between string theory in this AdS slice and a confining gauge theory on the boundary. The string world-sheet area would be dual to the static gauge theory Wilson loop. The identification of the string energy with the Cornell potential fixes an effective AdS radius, the position of the infrared brane and the string tension.

Recent results in the literature also support the idea of this approximate duality. An AdS slice, or equivalently an AdS space with an infrared cut off, was used in refs. [15, 16, 33, 34, 35, 36, 37] to discuss hadronic scattering from a string point of view. It has also been used to obtain Regge trajectories and hadron masses [17, 18, 19] and masses, decay rates and couplings for the lightest mesons[38].

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